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Departure Trajectories for Interplanterary Vehicles

by W. E. Moeckel

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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION

REPORT

DEPARTURE TRAJECTORIES FOR INTERPLANETARY VEHICLES

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ABSTRACT

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Results indicate that launching can take place from any point on the earth's surface with velocity penalties not greater than that due to loss of the benefit of the earth's rotational component. Penalties associated

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SUMMARY

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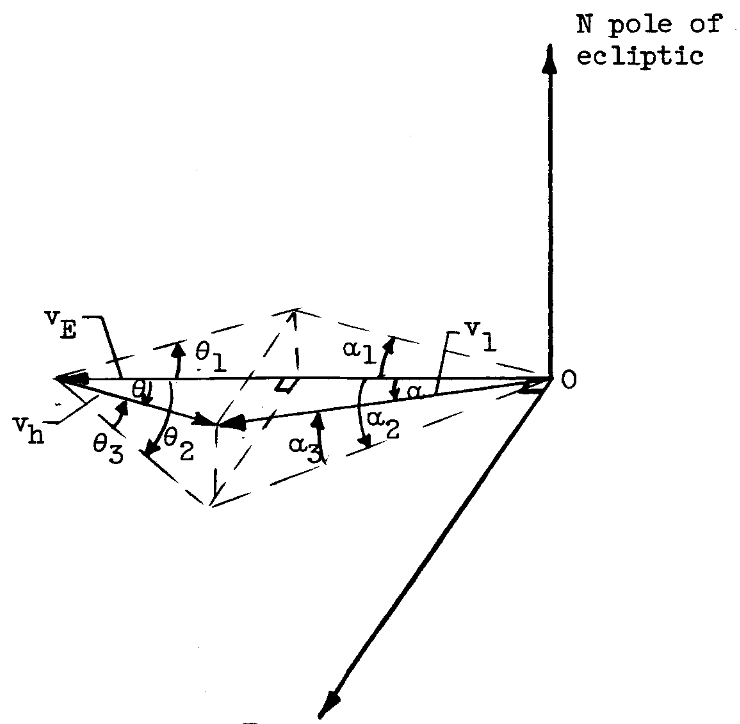
INTRODUCTION

The problem of determining the correct magnitude and direction of the launching velocity required to reach another planet from a given point on the earth's surface at a given time is a fairly complicated one. Solution requires detailed consideration of (1) the effect of launching time and the inclination of the orbital plane of the destination planet on the required magnitude and direction of the hyperbolic velocity vector relative to the earth, and (2) the effect of launch site and launch time on the initial velocity required to attain these hyperbolic velocities. In the present paper, general expressions are derived relating these parameters, and the results are applied to particular Earth-Venus trajectories.

RELATIONS BETWEEN HELIOCENTRIC AND HYPERBOLIC VELOCITY



Sketch 1

Shown in sketch 1 is the general relationship between the heliocentric velocity, $\underline{v_1}$, required at the earth's orbit to reach the destination, and the hyperbolic velocity, $\underline{v_h}$, required to attain this value of $\underline{v_1}$. The line to the sun and the earth's orbital velocity vector $\underline{v_E}$ determine the ecliptic plane. The angles α and θ are the inclination of the heliocentric velocity and the hyperbolic velocity relative to the orbital velocity $\underline{v_E}$: The angles α_1 and θ_1 are the inclinations of $\underline{v_1}$ and $\underline{v_h}$ northward from the ecliptic, and the angles α_2 and θ_2 are the inclinations of $\underline{v_1}$ and $\underline{v_h}$ toward the sun from $\underline{v_E}$. The angles α_3 and θ_3 are the northward inclinations of $\underline{v_1}$ and $\underline{v_h}$ with plane normal to the ecliptic. If the destination lies in the ecliptic plane, α_1 and θ_1 are zero, and if the transfer trajectory is tangent to the earth's orbit, α_2 and θ_2 are zero.

In the sketch $\underline{v_1}$ is shown as less than $\underline{v_E}$, as it might be for trajectories to the innerplanets. In general, the magnitude of $\underline{v_1}$ and its angle α_2 relative to $\underline{v_E}$ are determined from the co-planar problem, i.e., if there is no midcourse correction, the destination must lie in the plane determined by $\underline{v_1}$ and the line to the sun O-S. Thus, $\underline{v_1}$ and α_2 can be considered known functions of launching time. The inclination of the trajectory plane, α_1, relative to the ecliptic plane, can also be calculated as function of the position of the points of departure and arrival, as shown below.

From sketch 1, with v_1, α_1 and α_2 known, the remaining parameters may be calculated from the following equations:

$$\cos^2\alpha = \frac{\cos^2\alpha_1 \cos^2\alpha_2}{\cos^2\alpha_1 + \cos^2\alpha_2 - \cos^2\alpha_1 \cos^2\alpha_2} \tag{1}$$

$$v_h^2 = (v_E - v_1\cos\alpha)^2 + v_1^2\sin^2\alpha \tag{2}$$

$$\sin\theta = \frac{v_1}{v_h}\sin\alpha \tag{3}$$

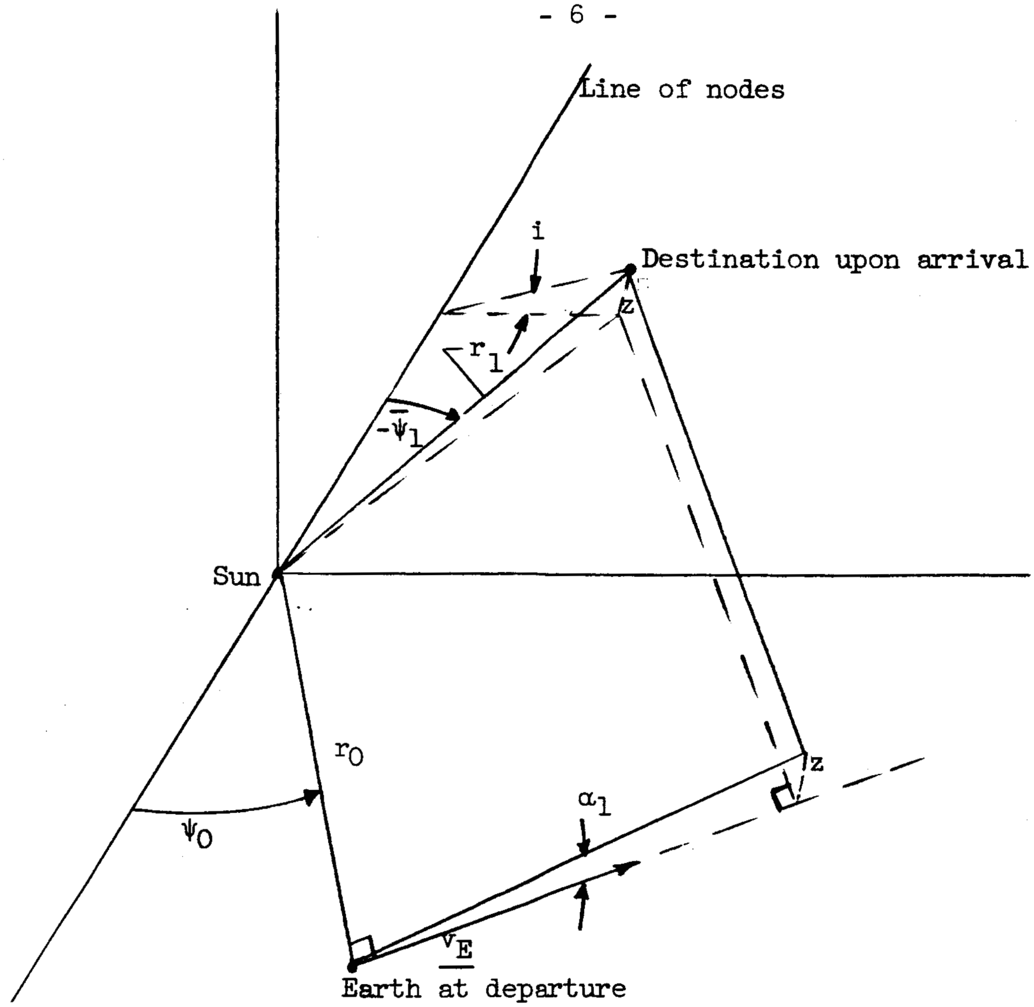
$$\tan\theta_1 = \frac{v_1\cos\alpha}{v_h\cos\theta}\tan\alpha_1 \tag{4}$$

$$\tan\theta_2 = \frac{v_1\cos\alpha}{v_h\cos\theta}\tan\alpha_2 \tag{5}$$

$$\cos\theta_3 = \frac{\cos\theta}{\cos\theta_2} \tag{6}$$

$$\cos\alpha_3 = \frac{\cos\alpha}{\cos\alpha_2} \tag{7}$$

<u>Determination of α_1:</u> The angle α_1, which is the inclination of the plane containing the sun, the earth at departure, and the destination at arrival, can be determined as function of time with the aid of sketch 2. Let ψ_0 be the angular distance of the earth from the line of nodes at departure and ψ_1 the angular distance of the destination planet from the line of nodes upon arrival of the vehicle, both angles being measured in the direction of motion of the planets. The angle i is the inclination



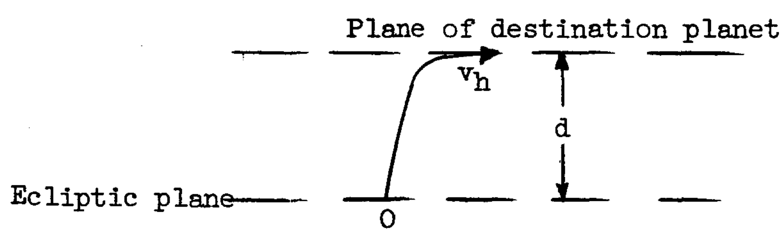
Sketch 2

of the orbital plane of the destination planet. Some trigonometric manipulation shows that for $\sin^2 i \ll 1$,

$$\tan \alpha_1 = \frac{-\sin i \sin \psi_1}{\cos \psi_0 \sin \psi_1 - \sin \psi_0 \cos \psi_1} \tag{8}$$

This relation shows, as might be expected, that α_1 is 90° if $\psi_0 = \psi_1$, except when $\psi_1 = 0$. In other words, when departure and destination points are 180° apart, the only plane containing both of these points and the sun is perpendicular to the ecliptic. This trajectory, of course, is prohibitively expensive in terms of velocity required, since the orbital motion of the earth must be completely canceled, and a velocity v_1 perpendicular to the earth's orbit must be provided. Obviously, other methods of reaching the destination when $\psi_0 = \psi_1$ will require less energy.

Alternative methods include: (1) launching when the earth crosses the nodal line ($\psi_0 = 0$), at which time, from equation (1), $\alpha_1 = -i$, (2) timing the arrival to coincide with a nodal passage of the destination planet ($\psi_1 = 0$, $\alpha_1 = 0$); or (3) launching directly into the orbital plane of the destination planet. The third alternative, as shown in sketch 3,



Sketch 3

requires fairly high launch velocity unless the earth is very close to a node. The distance between the ecliptic plane and the plane of the destination planet is given by

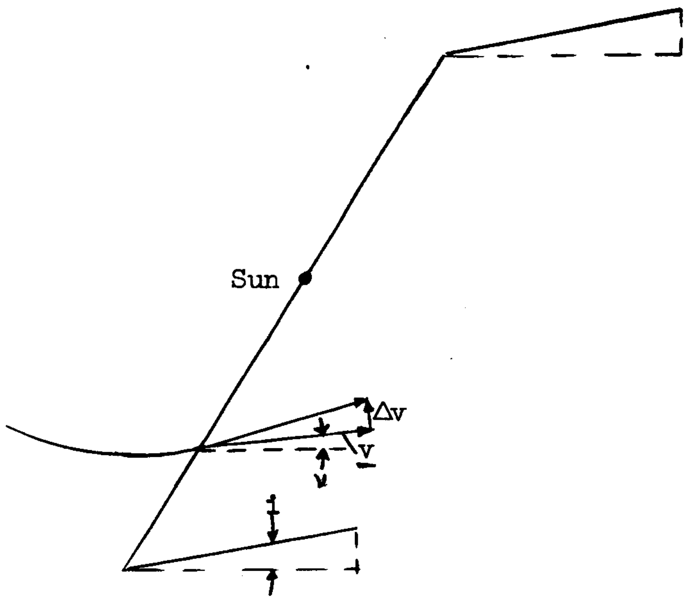
$$d = R_0 \sin i \sin \psi_0 \tag{9}$$

where R_0 is the distance from Earth to Sun. Unless $\psi_0 \approx 0$, d will be sufficiently large that the launch velocity to reach d must be very close to escape velocity. The required hyperbolic velocity must then be provided with an additional application of thrust when the distance d is reached.

The procedure in sketch 3 requires thrust application at fairly large distances from the earth. If midcourse corrective thrust is provided, other possibilities exist. For example, if the departure and arrival points are on opposite sides of the nodal line, an impulse can be provided, when the vehicle reaches the nodal line, which transfers the trajectory to the orbit plane of the destination planet. The impulse needed depends on the angle and velocity with which the vehicle approaches the nodal line (sketch 4). Thus

$$\frac{\Delta v}{v} = \sin i \cos \nu \tag{10}$$

where ν is the angle of approach relative to the normal to the nodal line.

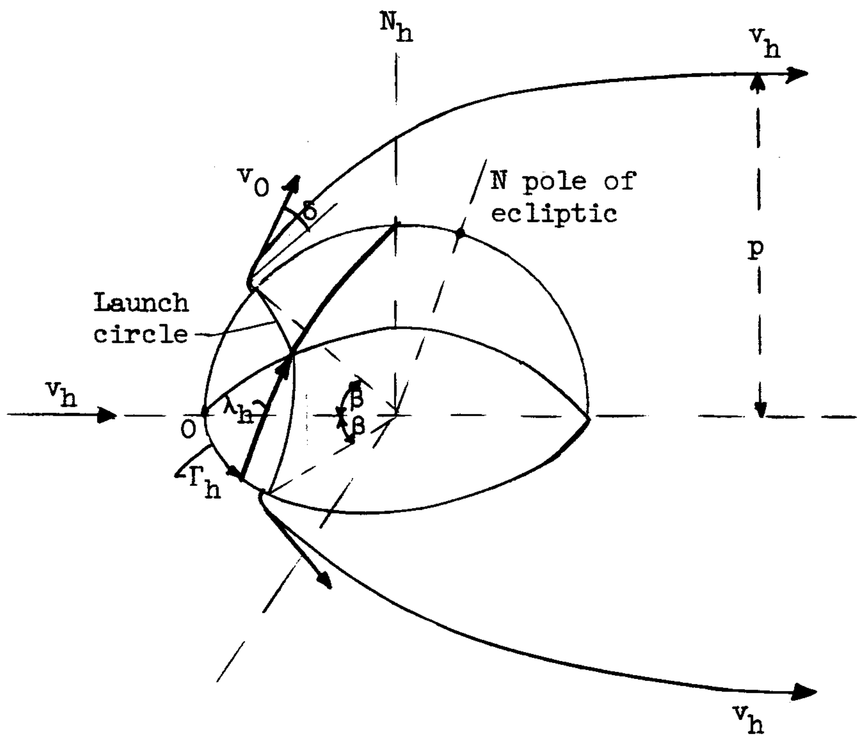


Sketch 4

Since ν is not far from zero for most trajectories, and v is of the order of 20 miles/sec, Δv is of the order of 1/2 to over 1 mile per second for Venus and Mars trajectories. In general, it is necessary to calculate the Δv required at many points along the trajectory to determine when a midcourse correction is minimized. One may conclude, however, that the most promising methods of allowing for the inclination of the orbital plane of the destination planet are alternatives (1) and (2) above, or failing this, choosing a trajectory for which ψ_1 and ψ_0 are sufficiently far apart so that α_1 (equation (8) is relatively small. Such trajectories, of course, are excess energy paths in terms of coplanar orbits, so that an optimum, or minimum-energy, trajectory exists which is different from a Hohman ellipse and requires, in general, higher launch velocity.

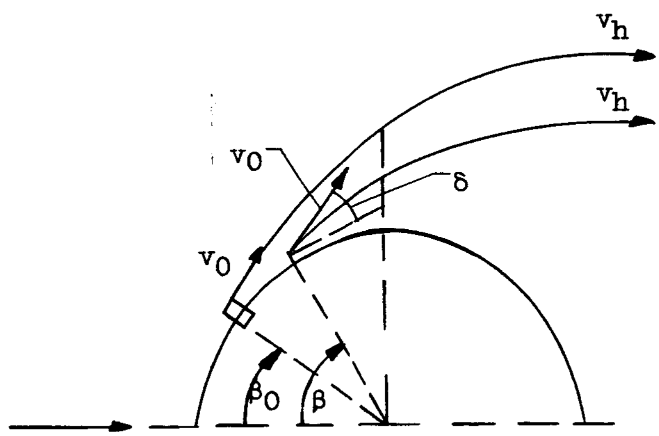
Earth Launch Conditions to Attain Specified Hyperbolic Velocity

The hyperbolic velocity vector required to produce the heliocentric velocity $\underline{v_1}$ can be achieved by launching from points on the earth's surface which lie on a circular cone whose axis passes through the earth's center and is parallel to $\underline{v}_h$, and whose half-angle is β, where β is the great-circle angle from the surface intersection of the diameter parallel to $\underline{v}_h$ to the launch point (sketch 5). This coordinate system



Sketch 5

will be denoted as the "hyperbolic" system, in which the north pole N_h is arbitrarily taken to be latitude $\lambda_h = 90^\circ$ along the great circle that contains the pole of the ecliptic plane. The latitude is measured northward from the equator, and longitude along the equator counterclockwise from 0. From each point on the launch circle, the same hyperbola is followed to achieve v_h. As β is increased larger inclinations δ of the launch velocity vector v_0 relative to horizontal must be provided to attain v_h. A minimum value, β_0, of β exists (see sketch 6) for which v_0 is horizontal. For β less than β_0, launch angle δ would be negative. The mathematical relations between v_h, β, v_0, and δ are obtained as follows:

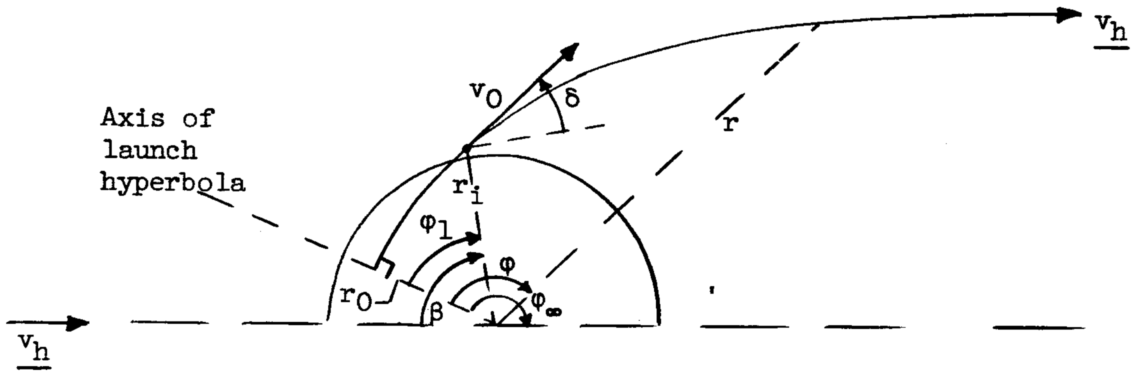


Sketch 6

From the energy equation

$$v_0^2 - 2\frac{\mu}{r_1} = v_h^2 \tag{11}$$

so that the final launch velocity is independent of β if it is always obtained at the same radius r_1. Only the inclination δ changes. However, the velocity v_0 is the resultant of the local


Axis of launch hyperbola
v_h
v_0
δ
r
r_i
φ_1
φ
φ_∞
r_0
β
v_h

Sketch 7

component of the earth's rotational velocity and the velocity $\underline{v_L}$ that must be provided by the launch motors. Consequently, v_L changes as β increases, but the precise nature of this change depends on the relationship between the "hyperbolic" coordinate system and the terrestrial system, which will be derived later.

To determine δ as a function of β, we utilize the general equation for a conic-section trajectory:

$$r = \frac{h^2/\mu}{1 + \epsilon \cos \varphi} \tag{12}$$

where h is angular momentum, μ the gravitational constant for the Earth, ϵ the eccentricity, and φ the trajectory angle measured from the axis of the hyperbola (see sketch 7). It is easily shown (see, for example, ref. 1) that

$$\epsilon = \frac{v_{00}^2}{\mu/r_0} - 1 = \frac{v_{00}^2}{v_{c00}^2} - 1 \tag{13}$$

where r_0, v_{c00}, and v_{00} are the radius, circular velocity and actual velocity at the axis of the hyperbola ($\varphi = 0$). Consequently,

$$\frac{r_1}{r_0} = \frac{v_{00}^2}{\frac{\mu}{r_0} + \left(v_{00}^2 - \frac{\mu}{r_0}\right)\cos \varphi_1} = \frac{v_h^2 + 2\mu/r_0}{\frac{\mu}{r_0} + \left(v_h^2 + \frac{\mu}{r_0}\right)\cos \varphi_1} \tag{14}$$

where the third term is obtained by application of the energy equation. The inclination δ is given by (see ref. 1)

$$\tan \delta = \frac{\sqrt{\mu/r_0}}{v_{00}} \sqrt{\left(\frac{r_1}{r_0}\right)^2 \left(\frac{v_{00}^2}{\mu/r_0} - 2\right) + 2\,\frac{r_1}{r_0} - \frac{v_{00}^2}{\mu/r_0}}$$

$$= \sqrt{\frac{v_h^2 \left[\left(\frac{r_1}{r_0}\right)^2 - 1\right] + 2\,\frac{\mu}{r_0}\left(\frac{r_1}{r_0}\right) - 1}{v_h^2 + \partial\,\mu/r_0}} \qquad (15)$$

To determine δ, with v_h and r_1 specified, we must, therefore, determine r_0 (or v_{c00}) as function of β. We note first from sketch 7 that

$$\beta + (\varphi_\infty - \varphi_1) = 180^\circ \qquad (16)$$

where φ_∞ is the value of φ when $r \to \infty$. From equation (12), this value is

$$\cos \varphi_\infty = -\frac{1}{\epsilon} = \frac{-\mu/r_0}{v_h^2 + \mu/r_0} \qquad (17)$$

Substituting φ_1 from equation (16) into equation (14), with equation (13) to eliminate φ_∞, we obtain an equation expressing r_0 as function of β for any value of v_h^2 and r_1. The expression for β_0 is obtained by setting $\varphi_1 = 0$ $(r_1/r_0 = 1)$ in equation (16):

$$\cos \beta_0 = \cos (180 - \varphi_\infty) = -\cos \varphi_\infty = \frac{v_{c00}^2}{v_h^2 + v_{c00}^2} \tag{18}$$

Two other relations are useful before proceding to transformation of co-ordinate systems, namely, the distance of the asymptote from the earth's axis (sketch 5) and the equation for the launch circle. From conservation of angular momentum,

$$p v_h = r_0 v_{00}$$

or

$$\frac{p}{r_0} = \sqrt{1 + 2\frac{v_{c00}^2}{v_h^2}} \tag{19}$$

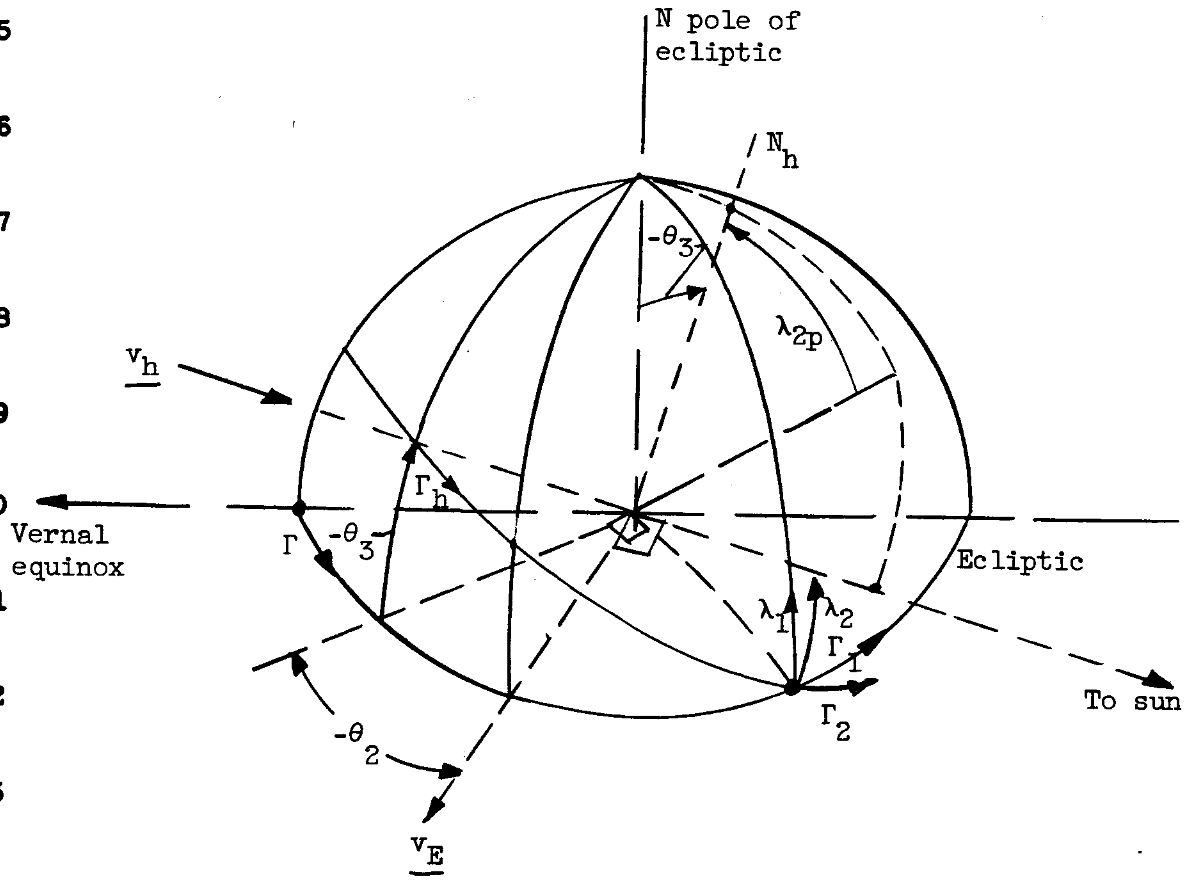
and from geometry:

$$\cos \lambda_h \cos \Gamma_h = \cos \beta \tag{20}$$

Equation (19) shows that the asymptote is normally within a few Earth radii of the axis of the hyperbola.

Relationship Between Coordinate Systems

In the celestial coordinate system, the intersection of the ecliptic plane with the earth is the equator, and the direction of the vernal equinox is the origin of coordinates. The relationship between this



Sketch 8

coordinate system and the hyperbolic velocity is shown in sketch 8. The general relationship between latitude and longitude in two great-circle systems is as follows (see appendix A).

$$\cos \lambda_1 = \frac{\cos \lambda_2 \cos \Gamma_2}{\cos \Gamma_1} \tag{21}$$

$$\tan \Gamma_1 = - \frac{\cos \lambda_{2p} \tan \lambda_2}{\cos \Gamma_2} + \sin \lambda_{2p} \tan \Gamma_2 \tag{22}$$

where λ_{2p} is the latitude in system 2 of the pole of system 1, and Γ is measured counterclockwise from the intersection of the two equators (90° clockwise from great circle through v_h). Thus, if system 2 is the hyperbolic system, denoted with subscript h, and system 1 the ecliptic system (without subscripts), then

$$\left.\begin{array}{ll} \lambda_2 \equiv \lambda_h ;\ \Gamma_2 \equiv \Gamma_h - 90^{\circ} & \lambda_{2p} = 90 + \theta_3 \\ \lambda_1 \equiv \lambda ;\ \Gamma_1 \equiv \Gamma - 90^{\circ} - (\Gamma_{v_E} + \theta_2) & \end{array}\right\} \tag{23}$$

Hence,

$$\cos \lambda = \frac{\cos \lambda_h \cos (\Gamma_h - 90^{\circ})}{\cos (\Gamma - \Gamma_{v_E} - \theta_2 - 90^{\circ})} = \frac{\cos \lambda_h \sin \Gamma_h}{\sin (\Gamma - \Gamma_{v_E} - \theta_2)} \tag{24}$$

$$\tan (\Gamma - \Gamma_{v_E} - \theta_2 - 90^{\circ}) = \frac{-\cos (90 + \theta_3) \tan \lambda_h}{\cos (\Gamma_h - 90^{\circ})} + \sin (90 + \theta_3) \tan (\Gamma_h - 90^{\circ})$$

or

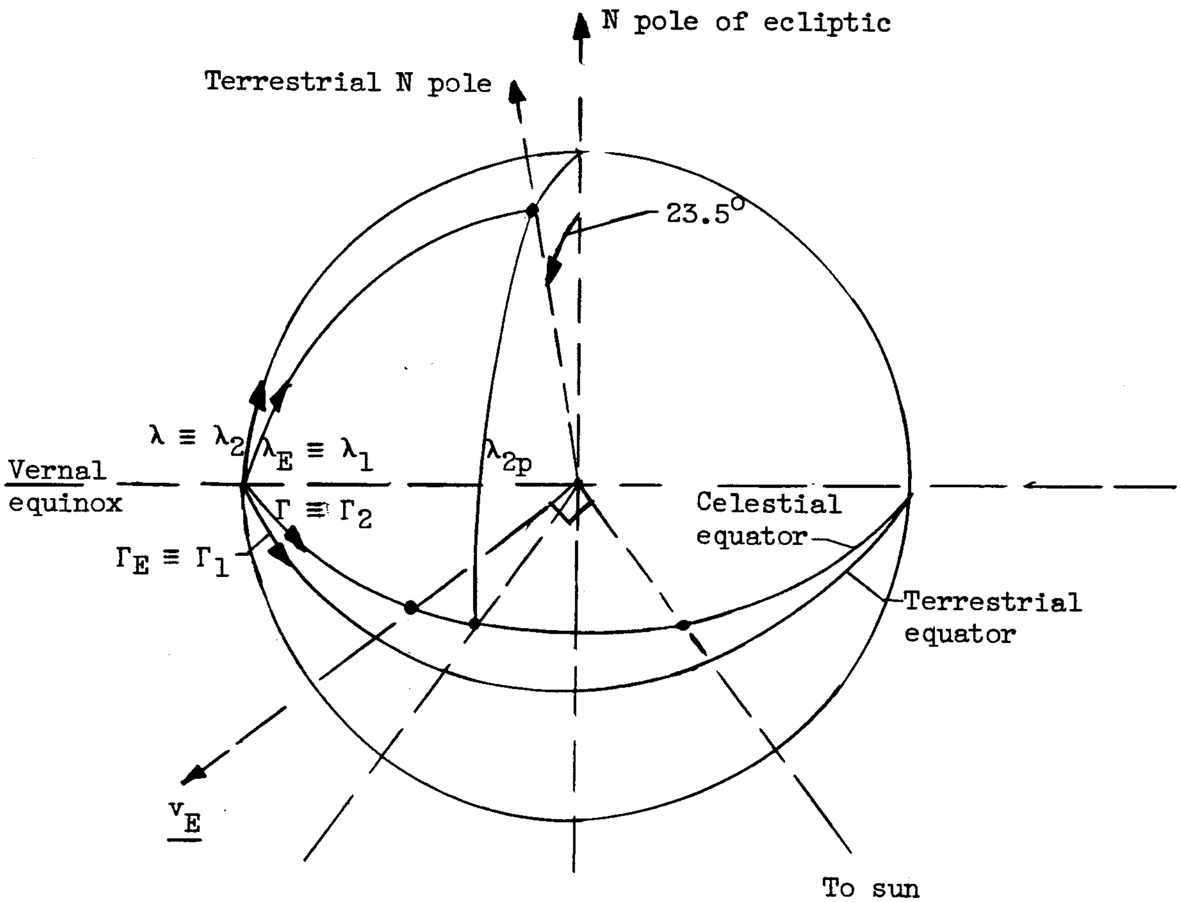
$$+ \cot (\Gamma_{v_2} + \theta_2 - \Gamma) = \frac{\sin \theta_3 \tan \lambda_h}{\sin \Gamma_h} - \cos \theta_3 \cot \Gamma_h \tag{25}$$

Equations (24) and (25), together with equation (20) permit calculations of the celestial latitude and longitude of the launching circle as function of β. The celestial longitude of $\underline{v_E}$ is

$$\Gamma_{v_E} = \Gamma_S - 90^\circ = \frac{360}{365}(t - t_{vE}) - 90^\circ \tag{26}$$

where t_{v_E} is the time of the vernal equinox (March 21), and Γ_S is the celestial longitude of the sun.

To convert to terrestial latitude and longitude (or hour angle), equations (21) and (22) are again applied; this time with system 1 being the terrestial system and system 2 the celestial, or ecliptic system. The coordinates are shown in sketch 9. Denoting celestial coordinate



Sketch 9

without subscript, and terrestial coordinates with subscript E, equations (21) and (22) yield:

$$\cos \lambda_E = \frac{\cos \lambda \cos \Gamma}{\cos \Gamma_E} \tag{27}$$

$$\tan \Gamma_E = \frac{-\sin 23.5 \tan \lambda}{\cos \Gamma} + \cos 23.5 \tan \Gamma \tag{28}$$

It remains only to determine the hour angle as function of Γ_E. The zero-point for the hour angle will be taken as local apparent noon, where the sun is at meridian. Thus, the hour angle γ after local noon is given by

$$\gamma = \Gamma_E - \Gamma_{ES} \tag{29}$$

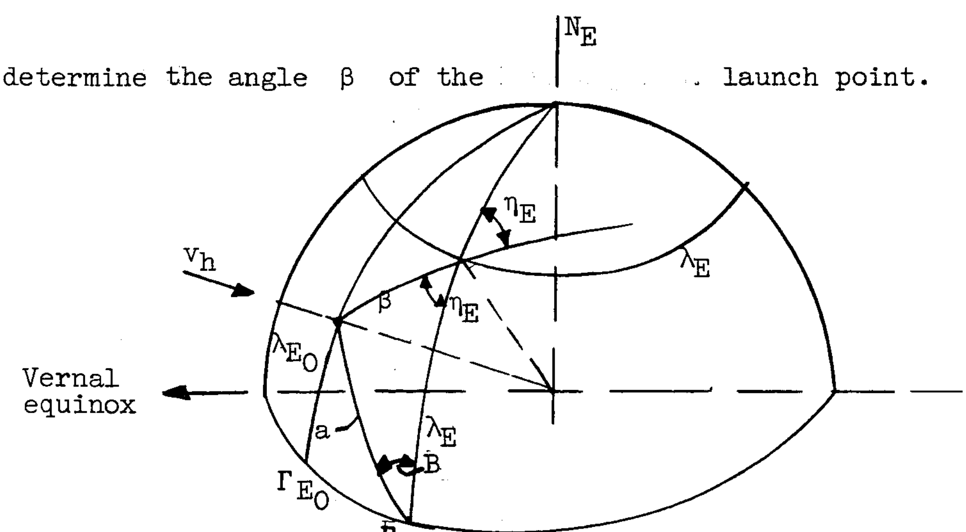
where, from equation (28) (with $\lambda = 0$)

$$\tan \Gamma_{ES} = \cos 23.5 \tan \Gamma_S \tag{30}$$

and Γ_S is given by equation (26).

Launch Azimuth and β

The launch azimuth and β can be calculated as functions of terrestial latitude and longitude with the aid of sketch 10. It is necessary first to determine the angle β of the launch point.



Sketch 10

From the cosine law of spherical trigonometry,

$$\cos \beta = \sin \lambda_E \sin \lambda_{EO} + \cos \lambda_E \cos \lambda_{EO} \cos (\Gamma_E - \Gamma_{EO}) \tag{31}$$

where λ_{EO} and Γ_{EO} are the terrestial latitude and longitude of the intersection of the diameter parallel to $\underline{v_h}$, with the earth's surface. These angles are obtained from sketch 8 and equation (27) and (28). Thus, in celestial coordinates, $\Gamma_0 = \Gamma_{v_E} + \theta_2$; $\lambda_0 = -\theta_3$, so that

$$\tan \Gamma_{EO} = \frac{+\sin 23.5 \tan \theta_3}{\cos (\Gamma_{vE} + \theta_2)} + \cos 23.5 \tan (\Gamma_{v_E} + \theta_2) \tag{32}$$

$$\cos \lambda_{EO} = \frac{\cos \theta_3 \cos (\Gamma_{v_E} + \theta_2)}{\cos \Gamma_{EO}} \tag{33}$$

The launch azimuth, η_E, is given by

$$\sin \eta_E = \frac{\sin a \sin B}{\sin \beta}$$

where a and B are obtained by the cosine law.

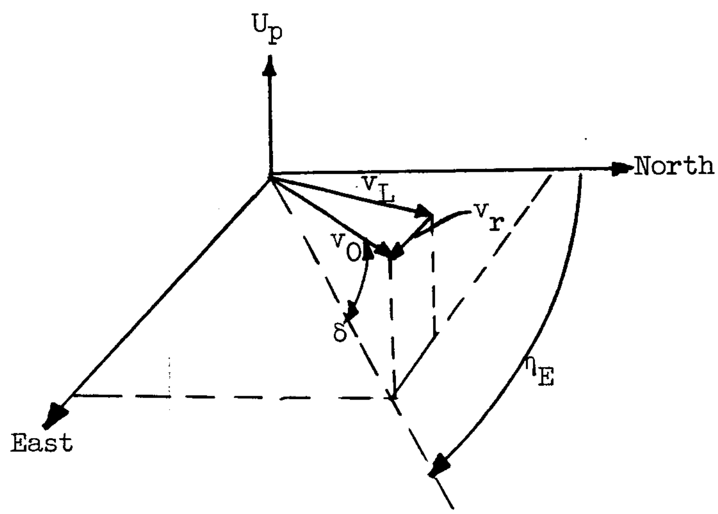
$$\cos a = \cos \lambda_{EO} \cos (\Gamma_E - \Gamma_{EO}) \tag{34}$$

$$\cos B = \frac{\sin \lambda_{EO}}{\sin a} \tag{35}$$

The resulting equation for η_E is

$$\sin \eta_E = \frac{\sqrt{1 - \sin^2\lambda_{EO} - \cos^2\lambda_{EO} \cos^2 (\Gamma_E - \Gamma_{EO})}}{\sin \beta} \tag{36}$$

The angle η_E, together with the inclination δ relative to horizontal, permits calculation of the orbital velocity v_L that must be provided by the launch motors when the earth's rotational velocity is considered. As shown in sketch 11, the three components of v_L are



Sketch 11

$$v_{L,up} = v_O \sin \delta$$

$$v_{L,north} = v_O \cos \delta \cos \eta_E \tag{37}$$

$$v_{L,east} = v_O \cos \delta \sin \eta_E - v_r$$

where v_r is the rotational speed of the earth at the launch latitude.

Thus, with $v_r = v_{rO} \cos \lambda_E$

$$v_L^2 = v_O^2 - 2v_O v_{rO} \cos \delta \sin \eta_E \cos \lambda_E + v_{rO}^2 \cos^2 \lambda_E \tag{38}$$

This equation, together with those previously derived relating δ, η_E,

λ_E and γ with the required hyperbolic velocity, permit calculation of the required launching velocity v_L as function of launching latitude and time of day for any chosen interplanetary trajectory. Equation (38) shows, as does sketch 10, that launching can be accomplished from any latitude on the earth with a maximum velocity penalty equal to loss of the benefit of the earth's rotational speed. The maximum benefit of this rotational speed occurs when $(\cos \delta \sin \eta_E \cos \lambda_E)$ is maximized, if launch latitude is arbitrary, or when $(\cos \delta \sin \eta_E)$ is maximized if launch latitude is fixed. The best launching latitude is, therefore, determined by the direction of the required hyperbolic velocity vector and is not necessarily the equator.

Application to Earth-Venus Trajectories

A convenient starting procedure to determine favorable launching periods is to consider the velocity increments needed as function of time on the basis of co-planar analysis. Many possible trajectories can be taken, but only two families of transfer ellipses are considered herein. Shown in figures 1 and 2 are the co-planar launch velocities needed to reach Venus and Mars, respectively, along these two families. The results were calculated from the data given in reference 1. The four curves correspond to following the long (L) or short (S) branches of ellipses tangent to the earth's orbit (E) and tangent to the Venus or Mars orbit (V or M). The Δv's shown are $v_0 - v_{c00}$, where v_{c00} is the circular velocity at the assumed launch radius of 1.1 times the radius of the Earth. (Possible benefits to be derived from the earth's rotation are not considered in these Δv's.) The orbits of the planets were assumed to be circular which, in the case of Mars, can result in errors of about ± 0.1 miles/sec in Δv. Also shown in figures 1 and 2 are the distance and angle between earth and planet as function of time. The departure and arrival patterns repeat themselves during each synodic period.

The crossings of the Venus-Earth nodal line are also indicated. As pointed out in an earlier section, the most convenient launch times, from the standpoint of allowing for inclination of the Venus orbital plane, are those for which departure takes place when Earth crosses the nodal line, or arrival occurs when Venus crosses the nodal line. Figure 1 shows that the June 7, 1959 minimum-energy launching is unique in that the Earth is crossing the nodal line at departure, and Venus is crossing the nodal line at arrival (Nov. 2, 1959). For this date, therefore, the vehicle could be launched directly either into the ecliptic plane or into the Venus orbital plane. During the following synodic period, a slightly excess energy trajectory along an E-S ellipse (launching about 2 days after minimum-energy) produces an arrival time about May 15, 1961, when Venus is crossing the nodal line (descending node). In general, during each synodic period, there exists a launch date and trajectory which either coincides at departure with an Earth node (June 7 and Dec. 7) or coincide at arrival with a Venus node. Table I showsthe range of departure and arrival dates, during the next five synodic periods, for which Δv_0 does not exceed 2.31 miles/sec (0.2 miles/sec above minimum-energy Δv_0). In three of these periods (first, second and fifth) an Earth crossing of the nodal line occurs during the departure period, and in all periods a Venus node occurs during the arrival period. In many cases, however, the required trajectory for these dates may be inconvenient from the standpoints of guidance and communication requirements, or duration of the trip; or departure times may be delayed for other reasons. It is, therefore, desirable to determine the amount of excess Δv required as function of launching data to allow for inclination of the Venus orbit. Such computations become rather involved, because many families of trajectories should be considered, in addition to the two shown in figures 1 and 2. For illustration purposes the computational procedure will be carried out for only one family of trajectories, namely, the EL family, and for the 1959 synodic period.

Since the co-planar E-L trajectories require resultant heliocentric velocities v_1 parallel to v_E (sketch 1) the angles α_2 and θ_2 are zero, and $\alpha_1 = \alpha_3 = \alpha$, $\theta_1 = \theta_3 = \theta$, where α is the inclination of v_1 north from ecliptic. Table II shows the velocities and angles calculated for these trajectories as function of departure days after June 7.

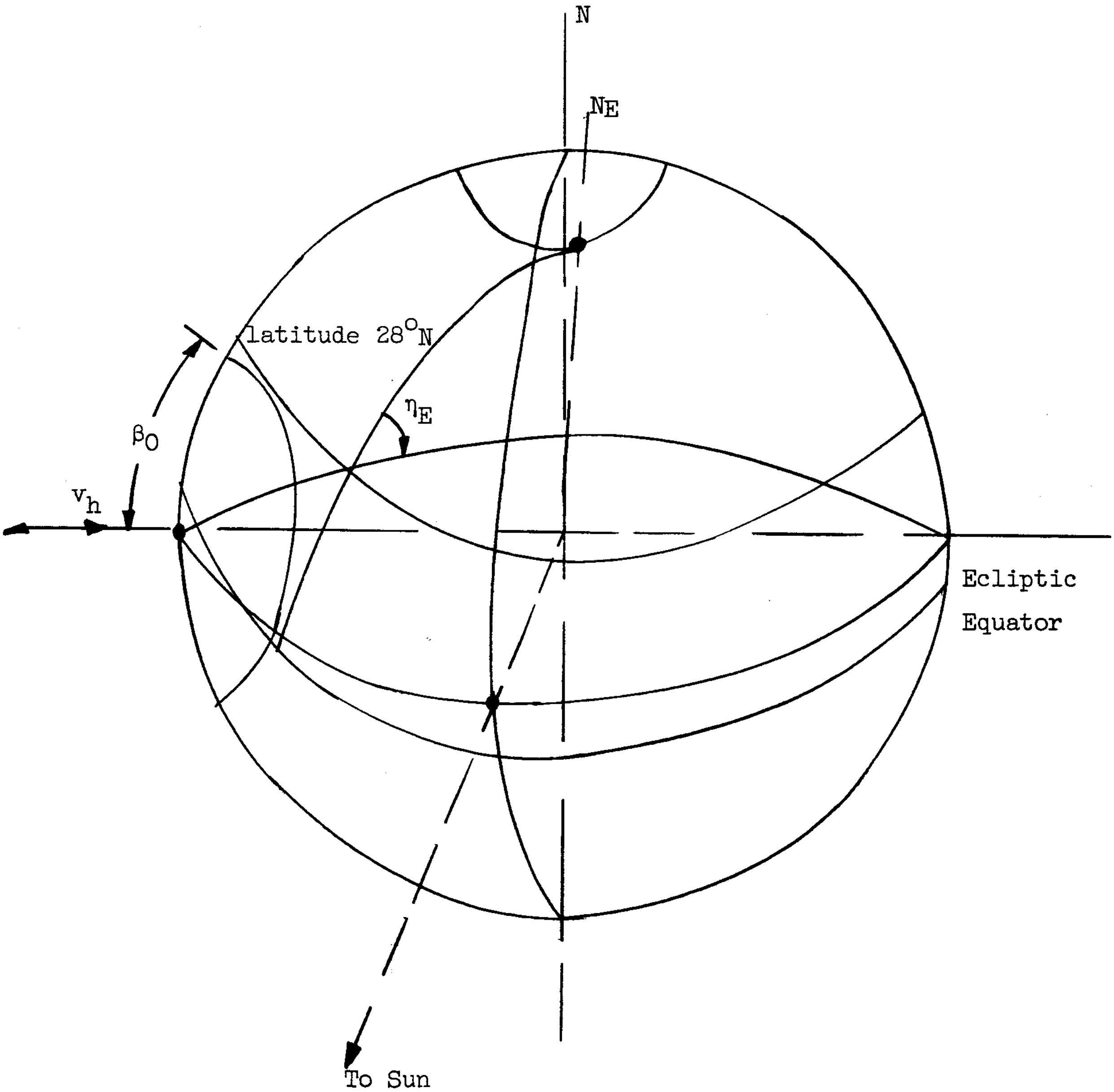
TABLE II. - VELOCITIES AND ANGLES FOR E-L VENUS TRAJECTORIES

Departure days after June 1	Arrival days after Nov. 2	v_1, $\frac{\text{mile}}{\text{sec}}$	α, deg	v_h, $\frac{\text{mile}}{\text{sec}}$	θ, deg	Δv_0, $\frac{\text{mile}}{\text{sec}}$	Δv, $\frac{\text{mile}}{\text{sec}}$	β_0, deg
0	0	16.95	0	1.56	0	2.12	2.12	25.5
1	2.4	16.95	-4.6	1.94	-44.5	2.12	2.22	31.4
5	12	16.92	-4.5	2.03	-39.4	2.13	2.27	32.6
20	41	16.74	-4.3	2.19	-35.2	2.18	2.30	34.6
45	70	16.41	-3.4	2.34	-24.6	2.27	2.34	36.8
60	85	16.19	-2.4	2.45	-16.2	2.33	2.38	38.2

The angle α was calculated from equation (8), v_h from equation (2), θ from equation (3), v_0 from equation (11), and β_0 from equation (18). The Δv's in this table are $v_0 - v_{c,0}$, with $v_{c,0}$ taken as 4.69 mile/sec, the value at a radius of 1.1 times the earth's radius. The velocity increment Δv_0 is the co-planar value, and Δv the value allowing for inclination of the Venus plane. These values are plotted in figure 1 for comparison (dashed curve). This plot shows that the effect of inclination on Δv is not large. However, if the minimum-energy launch had not coincided with a departure or arrival node, the angle α would be 90° at the "minimum energy" date, and Δv would have been extremely large indeed. A few days later, however, this penalty in Δv over co-planar values would be quite reasonable. In this case, the minimum-energy launch data, considering inclination, would be some days before or after the co-planar minimum-energy data.

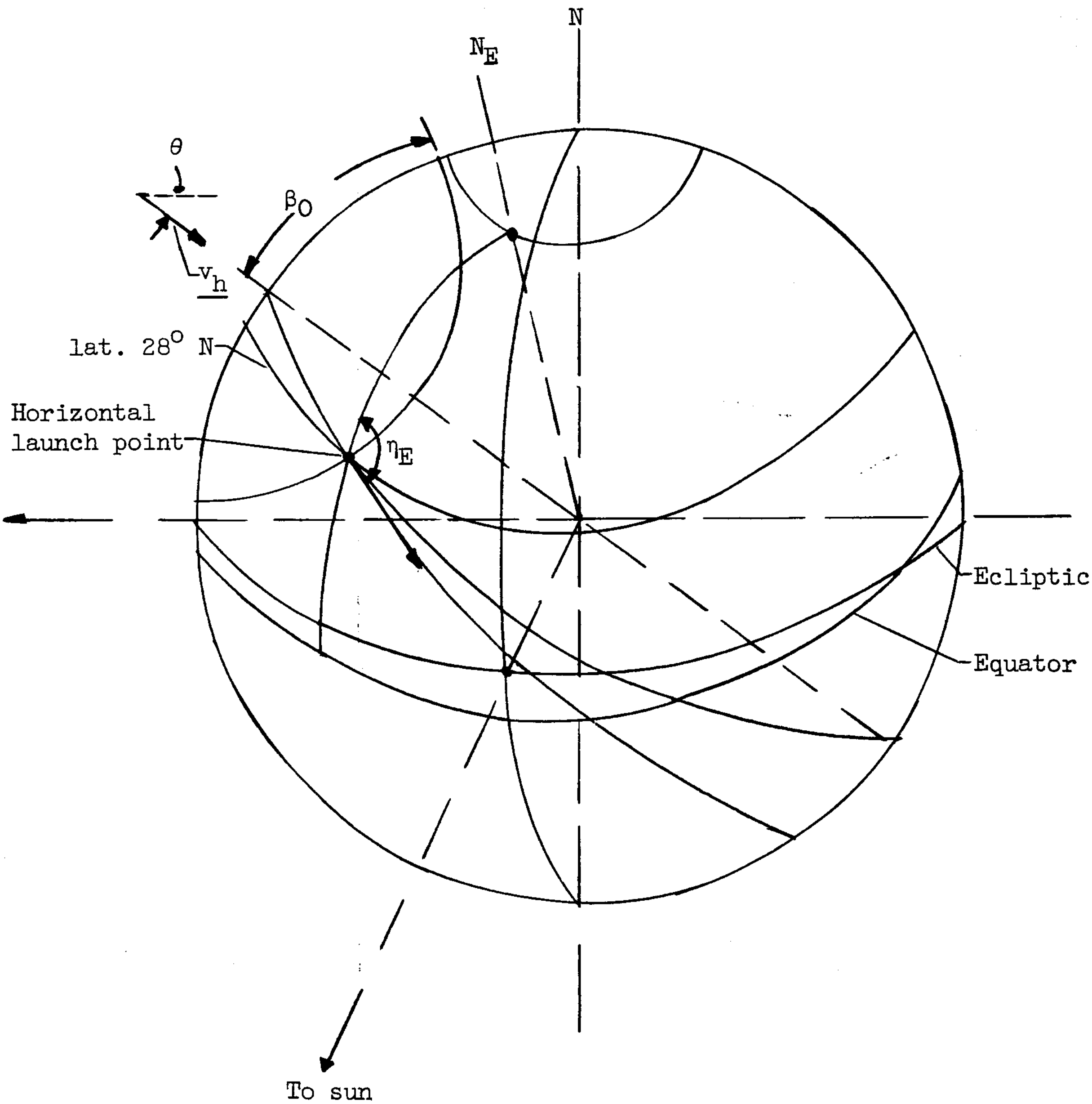
Table II shows that α and θ jump quite abruptly from zero to a maximum value, and then decline as the trajectory plane approaches the ecliptic plane. If the calculations were extended to about 90 days after June 7, figure 1 shows that α and θ would again be zero, since arrival would then coincide with another Venus node.

From the angles given in table II, the problem of launching from the earth's surface can be illustrated as in sketch 12 and 13.



No horizontal-launch point.

Sketch 12. - Launch situation June 7: $\theta = 0$; $\beta_0 = 25.5$.



Sketch 13. - Launch situation for E-L trajectory about 20 days after June 7: $\theta = -35.2^\circ$, $\beta_0 = 34.6$.

Sketch 12 shows the approximate location of the horizontal-launch cone for June 7, and sketch 13 for June 27. For the June 7 date, the latitude 28° N lies outside the horizontal-launch circle, so that the resultant velocity v_0 must have some inclination δ at all times during the launch day. To determine the best time of day for the launch, the penalty due to increasing δ must be balanced against the benefit due to increasing η_E (eq. 38 and sketch 11).

For the June 27 launch (sketch 13), latitude 28° N crosses the horizontal-launch circle at a time when the launch azimuth is not far from East, so that the crossing time would be a favorable launch time. Again, however, some benefit might result from waiting a little to produce even more easterly launch before δ has increased much.

The procedure to determine this optimum launch time is as follows: From equation 15, the expression for δ can be written

$$\tan \delta = \sqrt{\left(\frac{v_{c00}^2}{v_{c0}^2} - 1\right)\left[\frac{v_{c00}^2}{v_{c0}^2} + \frac{v_h^2 - 2v_{c00}^2}{v_h^2 + 2v_{c00}^2}\right]} \qquad (39)$$

where $v_{c00}^2 = \mu/r_0$, $v_{c0} = 4.69$; $v_h = 1.56$ for the June 7 launch and 2.19

for the June 27 launch. From equations (14) and (16):

$$-\cos(\beta + \varphi_\infty) = \frac{\left(\dfrac{v_h^2}{v_{c00}^2} + 2\right) - \dfrac{v_{c00}^2}{v_{c0}^2}}{\dfrac{v_h^2}{v_{c0}^2} + \dfrac{v_{c00}^2}{v_{c0}^2}} \tag{40}$$

where φ_∞ is given by

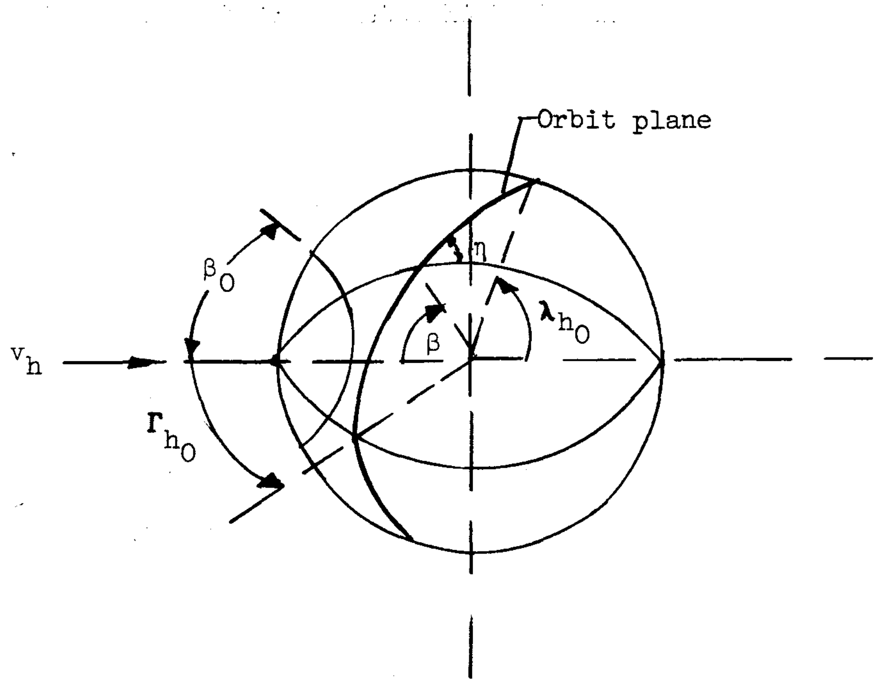
$$\cos \varphi_\infty = \frac{-v_{c00}^2}{v_h^2 + v_{c00}^2} = -\frac{1}{\dfrac{v_h^2}{v_{c00}^2} + 1} \tag{41}$$

By varying v_{c00}^2, δ is determined as functions of β. From equations (31), (32) and (33), Γ_E is determined as function of β. From equation (36), η_E is determined as function of Γ_E. Thus, η_E and δ are obtained as functions of Γ_E. The best value of Γ_E is that for which $\cos \delta \sin \eta_E$ is maximum (see eq. (38)). The hour angle corresponding to this Γ_E is obtained from equations (24) and (30).

Results of computations for launch from latitude 28° N along the E.L. path on June 27 are compared with results for the June 7 minimum-energy trajectory in figure 3. In both cases, the launch velocities required from the engines, v_L, can be very close to those that could be obtained if the full value of the earth's rotational speed at that latitude could be utilized. The launch azimuth and inclination, as might be expected, vary much more strongly with launch time than does the required launch velocity.

Launching from Satellite Orbits

The procedure described in the preceding sections for determining launching requirements from the Earth's surface is, of course, directly applicable to launching from the surface of any planet to return to Earth. Some modifications are necessary, however, for trips starting from satellite orbits. In this case the planet's rotation is no longer a factor, but the satellite velocity becomes an even more significant consideration. The problem of launching from circular and elliptic orbits has been discussed extensively in reference 2; only brief consideration will therefore be given herein.



Sketch 14

It is evident from sketch 14 that full advantage can be taken of satellite velocity of the vehicle only if the plane of the orbit contains the vector $\underline{v_h}$, or, in the "hyperbola" coordinate system, when $\Gamma_{h0} = 0$, where Γ_{h0} is the longitude of the nodal line of the orbital plane. In this case, the inclination of the orbital plane, λ_{h0}, is arbitrary. Launching can take place when the vehicle crosses the cone β_0, so that the added velocity is horizontal. If $\Gamma_{h0} \neq 0$, however, some perpendicular deflection of the trajectory, η, must be provided; furthermore if the orbit plane does not cross the β_0-cone, on upward inclination

δ must be provided. The actual launch velocity required is

$$v_L^2 = v_0^2 - 2v_0v_{c0} \cos\delta \cos\eta + v_{c0}^2 \tag{42}$$

where δ is again determined as function of β from equations (14), (15) and (16), and η is determined by spherical trigonometry from Γ_{h_0} and λ_{h_0}. It is evident from equation (42) that the penalty associated with large values of δ or η are much more severe than for surface launches, since $v_{c,0}$ is much greater than the rotational speed of the Earth. For this reason, orbital perturbations and precessions must be carefully precalculated if the vehicle is to remain in orbit for appreciable periods of time before departure. Furthermore, as pointed out in reference 2, difficulties may arise at the destination planet if the return vehicle remains in orbit. The vehicle may settle into an orbit with inclination as much as 90° relative to the departure direction, so that much of the saving in propellant associated with remaining in orbit rather than landing, may be negated. Reference 2, however, considers methods whereby the penalty due to deflecting the orbital plane may be reduced by applying the correction impulse after the vehicle has attained large distances from the planet where the vehicle velocity is much reduced.

CONCLUDING REMARKS

Analysis of the problem of launching interplanetary vehicles from the surface of a planet indicate that such launchings are possible from any latitude, with the maximum velocity penalty corresponding to loss of the benefit of the Earth's rotational speed. The maximum benefit of this rotational speed is derived when the product $\cos \delta \sin \eta_E \cos \lambda_E$ is maximized, where δ is the upward inclination of the launch velocity vector, η_E is launch azimuth, and λ_E is launch latitude. The angles δ and η_E as determined by the direction of the required hyperbolic velocity relative to the launch latitude.

The effect of inclination of the orbital plane of the destination planet is to change the required direction and magnitude of the hyperbolic velocity. The velocity penalty resulting from this inclination is generally quite small - of the order of 0.1 mile/sec for Earth-Venus trips - unless the departure and arrival points are nearly 180° apart and neither point is at a node. In the latter case, an excess-energy path, in the co-planar sense, may require considerably less velocity than the co-planar"minimum-energy" path.

Launching interplanetary vehicles from closed orbits around the departure planet may impose much more severe velocity penalties, relative to co-planar values, than launching from theplanet's surface, since the satellite velocity is much higher than the surface rotational speed. If the satellite orbital plane has significant inclination relative to the required hyperbolic velocity vector, much of the advantage associated with remaining in orbit rather than landing on a planet may be lost, unless the directional correction is applied at large distances from the planet where the relation vehicle velocity is small.

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AES-HAT/4-2-59

APPENDIX A

SYMBOLS

A,B,C	angles of spherical triangle
a,b,c	sides of spherical triangle
d	distance from ecliptic plane to orbital plane of destination planet
h	angular momentum
i	angle of inclination at orbital plane of destination planet
p	distance of asymtote of launch hyperbola from axis of launch cone
R	distance from Earth to destination planet
R_E	radius of Earth
R_0	distance from Earth to Sun
r	distance to trajectory from mass center
r_0	minimum distance of hyperbola from mass center
r_1	distance from mass center at which final launch velocity is attained
v	velocity along trajectory
Δv	$(v_0 - v_{c0})$ including inclination of orbital plane of destination

v_c circular (satellite) velocity

v_{c0} circular velocity at r_1

v_{c00} circular velocity at r_0

v_E Earth's orbital velocity

v_h hyperbolic velocity relative to Earth

v_L launch velocity provided by launch motors

v_r rotation speed of Earth's surface at launch latitude

v_{r0} rotation speed of Earth's surface at equator

v_0 resultant launch velocity at r_1

Δv_0 $(v_0 - v_{c0})$ from co-planar solution

v_{00} trajectory velocity at r_0

v_1 heliocentric velocity of vehicle at Earth orbit

α inclination of $\underline{v_1}$ to $\underline{v_E}$

α_1 inclination of plane of heliocentric trajectory north of ecliptic plane

α_2 inclination of v_1 in ecliptic plane

α_3 inclination of v_1 from ecliptic plane

β great-circle angle of launch point from origin of "hyperbolic" system

β_0 minimum value of β to attain v_h

Γ longitude in celestial coordinate system

Γ_E longitude in terrestial coordinate system

Γ_{E_0} terrestial longitude of radius parallel to $\underline{v_h}$

Γ_h longitude in hyperbolic coordinate system

Γ_s celestial longitude of sun

Γ_{v_E} celestial longitude of radius parallel to $\underline{v_E}$

γ hour angle, relative to local apparent noon

δ inclination of v_h from horizontal

ε eccentricity

η_E launch azimuth, counterclockwise from local north

θ inclination of $\underline{v_h}$ relative to $\underline{v_E}$

θ_M angle between Mars and Earth

θ_v angle between Venus and Earth

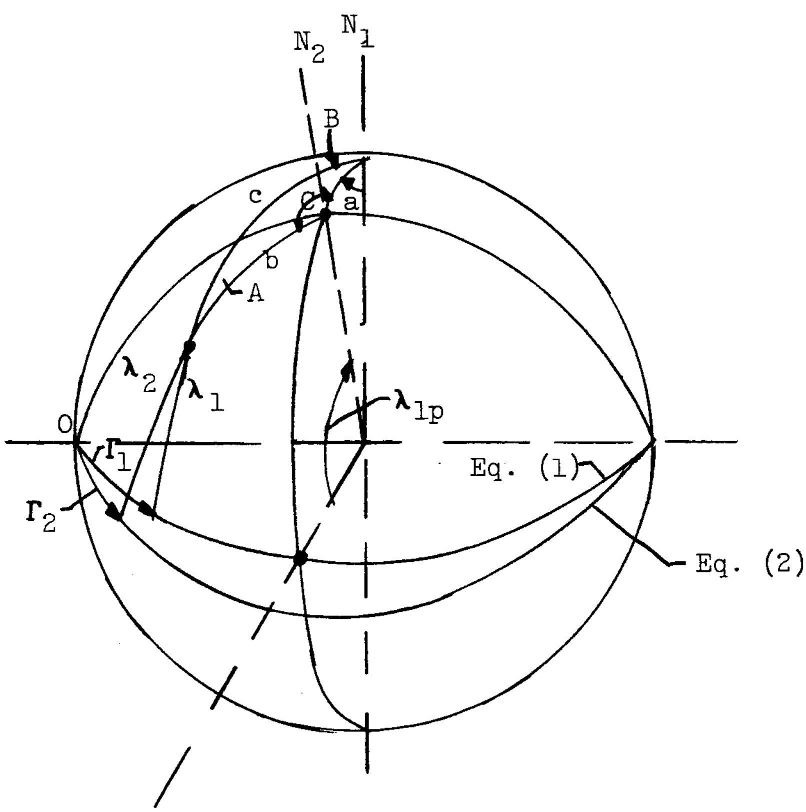
θ_1 inclination of plane of v_h north from ecliptic plane

θ_2	inclination of v_h in ecliptic plane
θ_3	inclination of v_h from ecliptic plane
λ	latitude in celestial coordiante system
λ_E	latitude in terrestial coordinate system
λ_{E_0}	terrestial latitude of radius parallel to v_E
λ_h	latitude in hyperbolic coordinate system
λ_s	celestial latitude of sun
λ_{v_E}	celestial latitude of radius parallel to v_E
μ	gravitational constant (9.6×10^4 miles^3/sec^2 for Earth)
ν	angle of intersection of vehicle path with nodal line
φ	trajectory angle, measured from axis of hyperbola
φ_∞	value of Φ for $r \rightarrow \infty$
φ_1	value of φ at r_1
ψ_0	angular distance of Earth from nodal line
ψ_1	angular distance of destination planet from nodal line

APPENDIX B

TRANSFORMATION OF COORDINATES

To transform from latitude and longitude in system 1 to latitude and longitude in another system 2 (sketch B-1), use is made of the conventional formulas of spherical trigonometry.



Sketch B-1

With the origin of longitude in both systems measured from the point of intersection of the two equators, the required formulas are

$$\frac{\sin A}{\sin a} = \frac{\sin B}{\sin b} = \frac{\sin C}{\sin c} \tag{B1}$$

$$\sin b \cos C = \sin a \cos c - \cos a \sin c \cos B \tag{B2}$$

where

$$\begin{aligned} a &= 90 - \lambda_{1p} & A &= A \\ b &= 90 - \lambda_2 & B &= 90 - \Gamma_1 \\ c &= 90 - \lambda_1 & C &= 90 + \Gamma_2 \end{aligned} \tag{B3}$$

Substitution of (B3) into (B1) and (B2) yields

$$\cos \lambda_2 \cos \Gamma_2 = \cos \lambda_1 \cos \Gamma_1 \tag{B4}$$

$$- \cos \lambda_2 \sin \Gamma_2 = \cos \lambda_{1p} \sin \lambda_1 - \sin \lambda_{1p} \cos \lambda_1 \sin \Gamma_1 \tag{B5}$$

These equations result directly in

$$\cos \lambda_2 = \frac{\cos \lambda_1 \cos \Gamma_1}{\cos \Gamma_2} \tag{B6}$$

$$\tan \Gamma_2 = - \cos \lambda_{1p} \frac{\tan \lambda_1}{\cos \Gamma_1} + \sin \lambda_{1p} \tan \Gamma_1 \tag{B7}$$

REFERENCES

1. Moeckel, W. E.: Interplanetary Trajectories with Excess Energy. Presented at Ninth International Astronautical Congress, Amsterdam, Aug. 23 to 30, 1958.

2. Bossart, Karel J.: Techniques for Departure and Return in Interplanetary Flight. Presented at 1958 National Midwestern Meeting, Institute of Aeronautical Sciences, St. Louis, May 14, 1953.

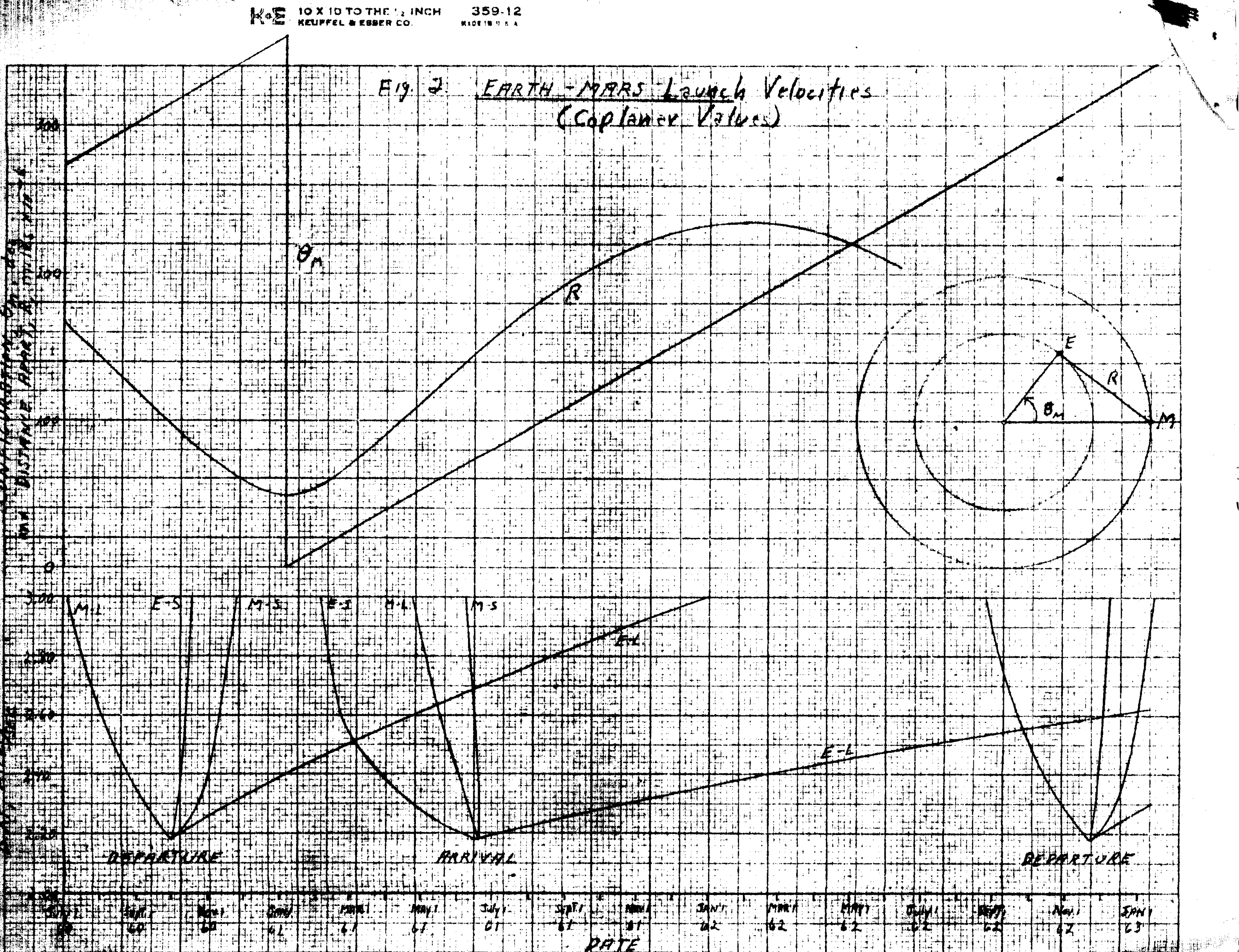
TABLE I

	1		2	
	Departure	Arrival	Departure	Arrival
Minimum energy	June 7, 1959	Nov. 2, 1959	Jan. 13, 1961	June 9, 1961
$\Delta v < 2.31$ Mile/sec	May 2, 1959 to Aug. 2, 1959	Sept. 20, 1959 to Jan. 22, 1960	Dec. 7, 1960 to Mar. 8, 1961	April 27, 1961 to Aug. 29, 1961
Venus node nearest arrival		Nov. 2, 1959 (ascending)		May 15, 1961 (descending)

	3		4	
	Departure	Arrival	Departure	Arrival
Minimum energy	Aug. 19, 1962	Jan. 17, 1963	Mar. 25, 1964	Aug. 23, 1964
$\Delta v < 2.31$ Mile/sec	July 14, 1962 to Oct. 14, 1962	Dec. 5, 1962 to April 5, 1963	Feb. 20, 1964 to May 20, 1964	July 11, 1964 to Nov. 11, 1964
Venus node nearest arrival		Mar. 19, 1963 (descending)		Sept. 30, 1964 (ascending)

	5	
	Departure	Arrival
Minimum energy	Nov. 2, 1965	Mar. 30, 1966
$\Delta v < 2.31$ Mile/sec	Sept. 28, 1965 to Dec. 28, 1965	Feb. 18, 1966 to June 18, 1966
Venus node nearest arrival		April 12, 1966 (descending)

Fig. 2 EARTH - MARS Launch Velocities
(Coplaner Values)



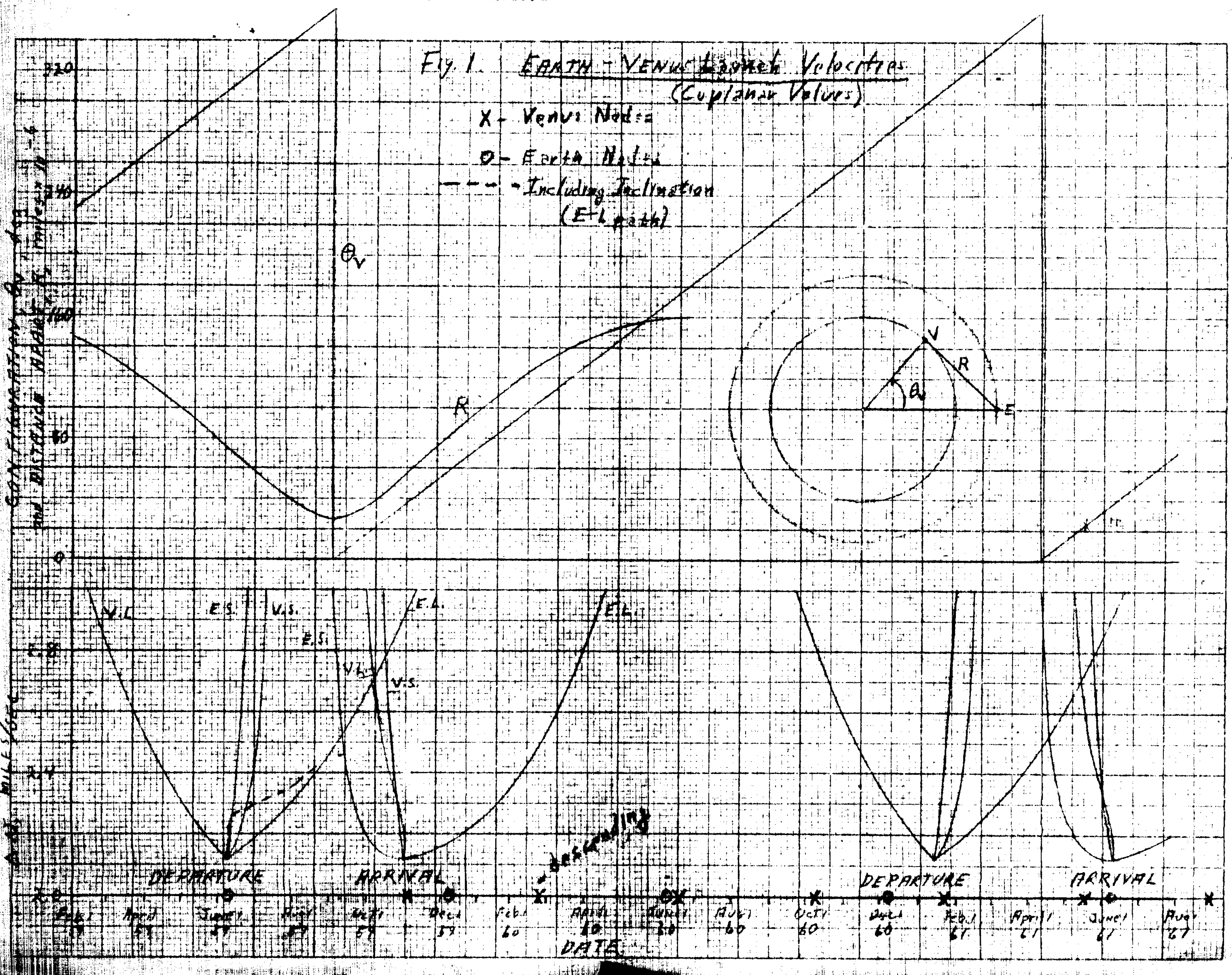


Fig. 1. Earth - Venus Launch Velocities
(Coplanar Values)
X - Venus Nodes
O - Earth Nodes
- - - Including Inclination (E-L path)
Configuration, θ_V, deg.
and distance apart, R, miles × 10^{-6}
miles/sec
θ_V
R
V
E
E.S.
V.S.
V.L.
E.L.
Departure
Arrival
Descending
Date
Feb. 59
April 59
June 59
Aug. 59
Oct. 59
Dec. 59
Feb. 60
April 60
June 60
Aug. 60
Oct. 60
Dec. 60
Feb. 61
April 61
June 61
Aug. 61

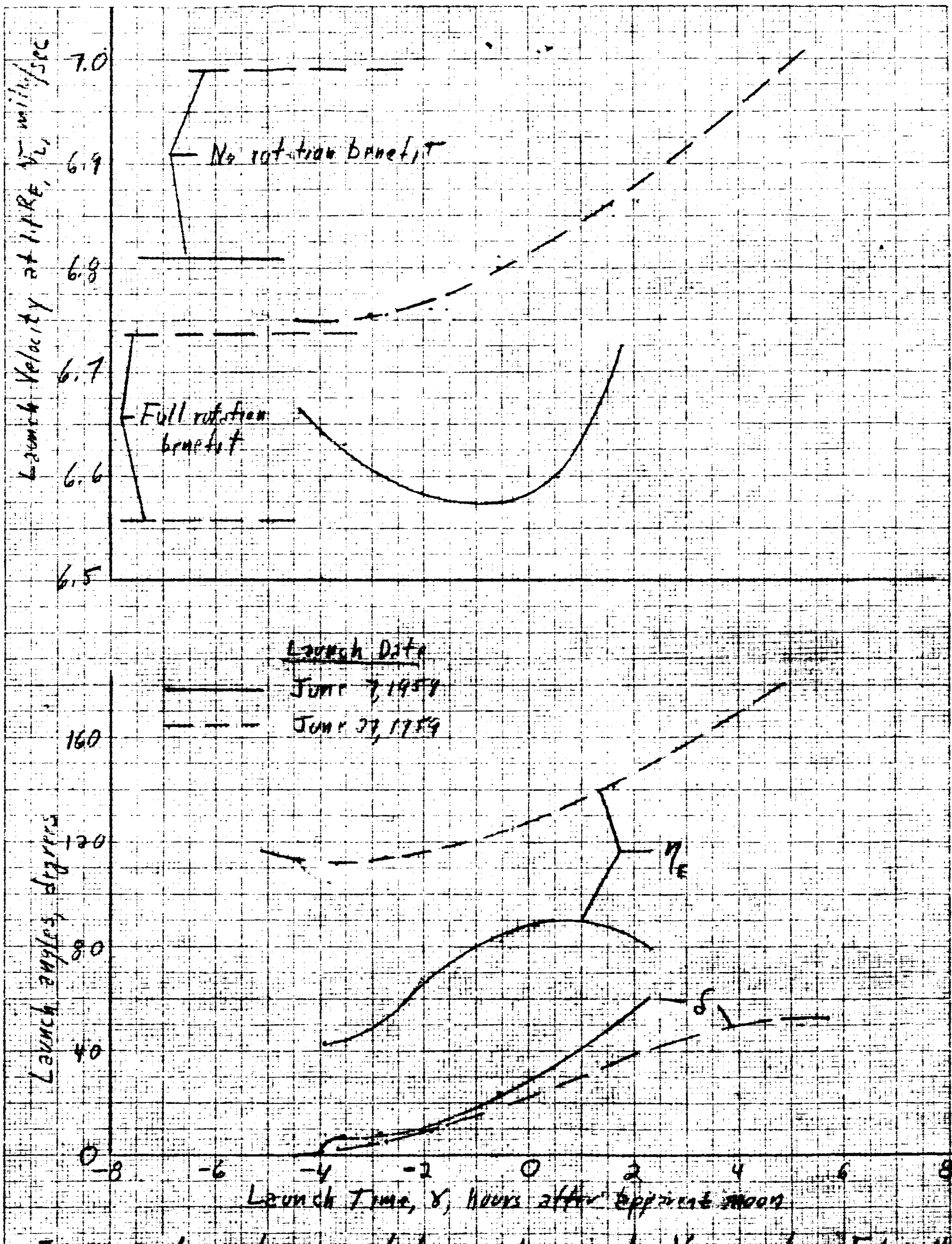


Figure 3. Launching conditions to reach Venus along E.L. path from latitude 28° N.